AQA

Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________

A-level
MATHEMATICS

Unit Further Pure 3

Wednesday 16 May 2018 Morning Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
TOTAL	

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Explain why $\int_1^{\infty} \frac{x-1}{e^x}\, dx$ is an improper integral.

[1 mark]

(b) Evaluate the improper integral $\int_1^{\infty} \frac{x-1}{e^x}\, dx$, showing the limiting process used.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 1

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QUESTION PART REFERENCE	Answer space for question 1

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2 It is given that $y(x)$ satisfies the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{f}(x, y)$$

where $$\mathrm{f}(x, y) = x + \frac{1}{2}\log_2(y + 7)$$

and $$y(2) = 1$$

Use the improved Euler formula

$$y_{r+1} = y_r + \frac{1}{2}(k_1 + k_2)$$

where $k_1 = h\mathrm{f}(x_r, y_r)$ and $k_2 = h\mathrm{f}(x_r + h, y_r + k_1)$ and $h = 0.3$, to obtain an approximation to $y(2.3)$, giving your answer to four significant figures.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 2

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QUESTION PART REFERENCE	**Answer space for question 2**

Turn over ▶

3 Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 10y = 34x - 20x^2$$

[7 marks]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE

Answer space for question 3

4 Show that, for some value of k,

$$\lim_{x \to 0}\left[\frac{3-\sqrt{9-kx^4}}{7x^6+8x^4}\right]=\frac{1}{32}$$

and state this value of k.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 4

QUESTION PART REFERENCE	Answer space for question 4

5 The polar equation of an ellipse C is

$$r = \frac{5}{3 + 2\sin\theta}, \qquad 0 \leqslant \theta \leqslant 2\pi$$

(a) By finding a Cartesian equation of the ellipse C in a suitable form, state the Cartesian equations of the tangents to C that are parallel to the coordinate axes.

[5 marks]

(b) Given that the area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is πab, deduce the exact value of $\int_0^{2\pi} \frac{1}{(3 + 2\sin\theta)^2}\, \mathrm{d}\theta$.

[4 marks]

QUESTION PART REFERENCE | **Answer space for question 5**

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QUESTION PART REFERENCE	Answer space for question 5

6 A second order differential equation is given by

$$\frac{d^2y}{dx^2} + (\cot x + \tan x)\frac{dy}{dx} = y\cot^2 x, \qquad 0 < x < \frac{\pi}{2}$$

(a) Show that the substitution

$$u = \frac{dy}{dx} + y\cot x$$

transforms the second order differential equation into

$$\frac{du}{dx} + u\tan x = 0$$

[3 marks]

(b) Hence, given that $y = 0$ and $\frac{dy}{dx} = \sqrt{3}$ when $x = \frac{\pi}{6}$, solve the second order differential equation to find an expression for y in terms of $\sin x$.

[9 marks]

QUESTION PART REFERENCE	**Answer space for question 6**

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QUESTION PART REFERENCE

Answer space for question 6

QUESTION PART REFERENCE	Answer space for question 6

7 (a) Find the general solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 4\sin 2x + 8\cos 2x$$

[6 marks]

(b) It is given that $y = f(x)$ is the solution of the differential equation

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 4\sin 2x + 8\cos 2x$$

such that the first two non-zero terms in the Maclaurin's series, in ascending powers of x, of $f(x)$ are $\frac{1}{2} + kx^2$. Find the value of $f\left(\frac{\pi}{6}\right)$, giving your answer in an exact form.

[4 marks]

QUESTION PART REFERENCE	**Answer space for question 7**

QUESTION PART REFERENCE	Answer space for question 7

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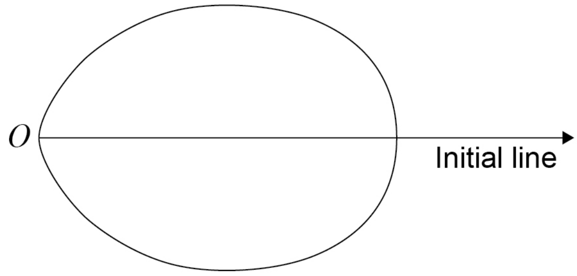
8 The diagram shows the sketch of a curve C_1.



The polar equation of the curve C_1 is $r = 4\cos^2\theta$, $-\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$.

The curve C_2 has polar equation $r = 1 + \tan\theta$, $0 \leqslant \theta < \frac{\pi}{2}$.

(a) Prove that the curves C_1 and C_2 intersect at a single point P whose distance from the pole O is 2.

[5 marks]

(b) The curve C_2 meets the initial line at the point A. Find the area of the region bounded by the line segment AP and the curve C_2, giving your answer in an exact form.

[8 marks]

QUESTION PART REFERENCE

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9 (a) Find the first three non-zero terms in the expansion of

$$\ln(1+y) - \ln(1-y)$$

in ascending powers of y **and** state the range of values of y for which the expansion is valid.

[3 marks]

(b) Use the identity $1 + x^3 \equiv (1+x)(1-x+x^2)$ to find the coefficient of x^{6r-3}, where r is a positive integer, in the expansion of

$$\ln\left(\frac{1-x+x^2}{1+x+x^2}\right)$$

in ascending powers of x. Give your answer in its simplest form.

[7 marks]

QUESTION PART REFERENCE	Answer space for question 9

QUESTION PART REFERENCE

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END OF QUESTIONS